Modelli per v.c. continue

	Task Description	Due Date	Status
☐	Variabili casuali continue: definizione, funzione di densità di probabilità, calcolo di probabilità relativo ad intervalli, principali indici di sintesi		
☐	Funzione di ripartizione e suo utilizzo per il calcolo di probabilità relativo ad intervalli di valori		
☐	Variabile casuale uniforme continua		
☐	Variabile casuale esponenziale negativa		
☐	Legame tra il modello esponenziale negativo e il modello di Poisson		
☐	La variabile casuale normale: definizione, caratteristiche, funzione di densità e proprietà (1/2)		
☐	Uso delle tavole della funzione di ripartizione della v.c. normale standard per il calcolo di probabilità relativo ad intervalli sottesi ad una generica v.c. normale		
☐			
☐	Calcolo dei percentili di una v.c. normale		
☐	Proprietà della v.c. normale (2/2): la proprietà riproduttiva		

RIEPILOGO

Modello di Bernoulli ⟶ codifica {0, 1} di un esperimento dicotomico

0 ⟶ insuccesso
1 ⟶ successo

Modello di Bernoulli ⟶ centrale per l'inferenza nel caso di variabili "qualitative"

V.C. LEGATE AL PROCESSO DI BERNOULLI

- modello binomiale ⟶ n° di successi in n ripetizioni di un esperimento Bernoulliano
- modello geometrico ⟶ n° prove necessarie ad ottenere il primo successo in ripetizioni successive di un esperimento di Bernoulli

caso particolare

- modello binomiale negativo ⟶ n° prove necessarie ad osservare i primi K successi in ripetizioni successive di un esperimento di Bernoulli

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- Poisson ⟶ n° eventi in un dato intervallo di tempo (o spaziale)

RIEPILOGO

F favorevoli
N − F non favorevoli

N palline

↓

n estrazioni ⇝ n° successi in n estrazioni successive

ESTRAZIONI IN BLOCCO — **CON REIMMISSIONE**

v.c. ipergeometrica ⇝ v.c. binomiale

se $N \to \infty$ i due schemi di estrazioni sono in pratica equivalenti

$X_i \sim Ber(\pi)$

$$S_n = \sum_{i=1}^{n} X_i$$

$S_n \sim bin(n, \pi)$

$$\boxed{\mu_{S_n} = n\pi \qquad \sigma^2_{S_n} = n\pi(1-\pi)}$$

ricorriamo alla trasformaz.

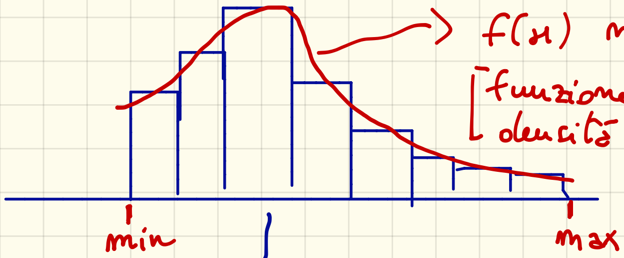
$$F_n = \frac{S_n}{n} = \frac{\sum X_i}{n} = \hat{\pi}$$

$$\boxed{\mu_{F_n} = \pi \quad e \quad \sigma^2_{F_n} = \frac{\pi(1-\pi)}{n}}$$

V.c. continue → assumono un'infinità di possibili valori

↓

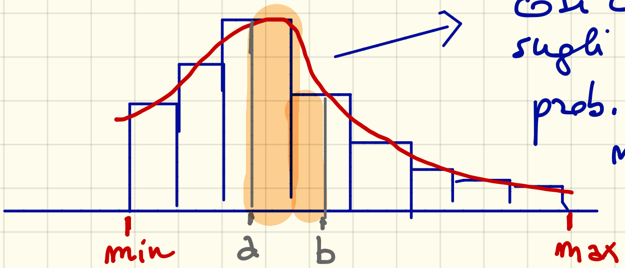
di misurazione



→ $f(x)$ mi descrive l'andamento della probabilità sull'insieme dei possibili valori che la v.c. X può assumere

[funzione di densità]

con l'istogramma descrivevamo l'andamento della frequenza (o della densità di frequenza) sull'intervallo dei possibili valori della variabile



così come visto con il ragionamento sugli istogrammi, per calcolare la prob. su un modello continuo è necessario misurare aree

INTEGRALE DEFINITO

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

In un punto non c'è area

$$\hookrightarrow P(X = x) = 0 = \int_x^x f(t)\,dt$$

$\forall x$

ev. quasi impossibili: l'evento $X = x$ si può verificare ma ha probab. $= 0$
(1 punto / ∞ possibili punti)

Nel caso di v.c. continue:

$$P(a \le X \le b) = P\{\underbrace{(X=a) \cup (a<X<b) \cup (X=b)}_{\text{ev. incompatibili (III assioma)}}\} =$$

$$= \underbrace{P(X=a)}_{=0} + P(a<X<b) + \underbrace{P(X=b)}_{=0}$$

Gli eventi:

$a \le X \le b$; $a < X < b$; $a \le X < b$; $a < X \le b$

sono diversi ma hanno tutti la stessa probabilità

Se X è una v.c. discreta, la sua distribuz. di probabilità

$$f(x) = P(X = x)$$

deve soddisfare due requisiti

$$f(x) \geq 0$$

$$\sum_x f(x) = 1$$

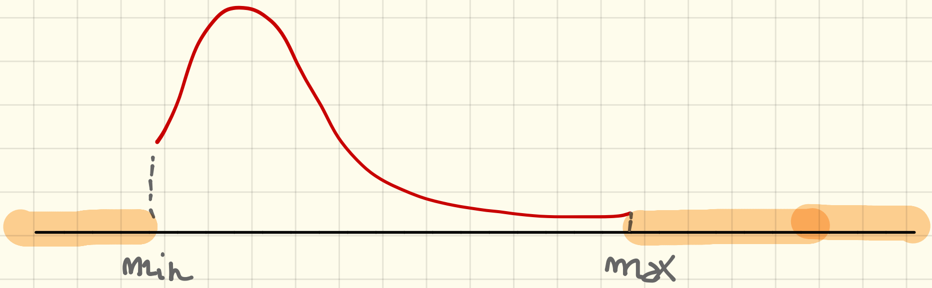
Se X è una v.c. continua, la sua funzione di densità

$f(x)$ $\longrightarrow$ può essere interpretata in termini di probab. in un intorno di x

deve soddisfare due requisiti:

- $f(x) \geq 0$
- $\int_{min}^{max} f(x)\,dx = P(min < X < max) = P(\Omega) = 1$

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1$$



$$\int_{-\infty}^{+\infty} f(x)\,dx = P(-\infty < X < +\infty) =$$

$$= P(-\infty < X < min) + P(min \le X \le max) + P(max < X < +\infty) =$$

$$\underbrace{\int_{-\infty}^{min} f(x)\,dx}_{=0} + \int_{min}^{max} f(x)\,dx + \underbrace{\int_{max}^{+\infty} f(x)\,dx}_{=0}$$

Indici di sintesi per v.c. continue

$$\mu_X \quad \begin{cases} \sum_x x\, p(x) = \sum_x x\, f(x) & \text{v.c. discrete} \\ \int_{-\infty}^{+\infty} x\, f(x)\, dx \end{cases}$$

$$\sigma_X^2 \quad \begin{cases} \sum_x (x-\mu_X)^2\, p(x) = \sum_x (x-\mu_X)^2 f(x) & \text{v.c. discrete} \\ \int_{-\infty}^{+\infty} (x-\mu_X)^2 f(x)\, dx \end{cases}$$

Funzione di ripartizione $\longrightarrow$ $F(x) = P(X \leq x)$

nella parte descrittiva abbiamo definito funzione di ripartizione empirica la frequenza cumulata

cumuliamo la probabilità fino al punto x

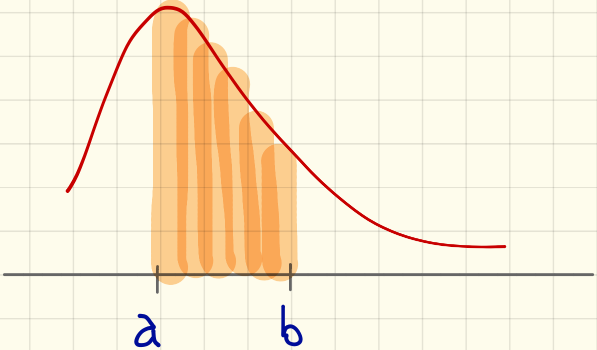
strumento comune sia ai modelli discreti che a quelli continui

NOTA BENE:

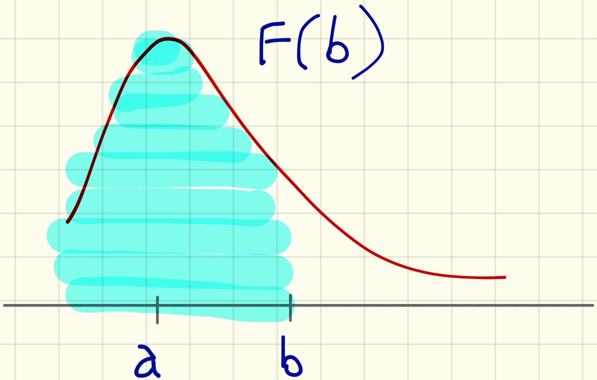
Se X è una v.c. $\quad F(x) = P(X \leq x) \equiv P(X < x)$

la $F(x)$ viene usata per calcolare la prob. in corrispondenza di un intervallo $]a, b]$ $\rightarrow$

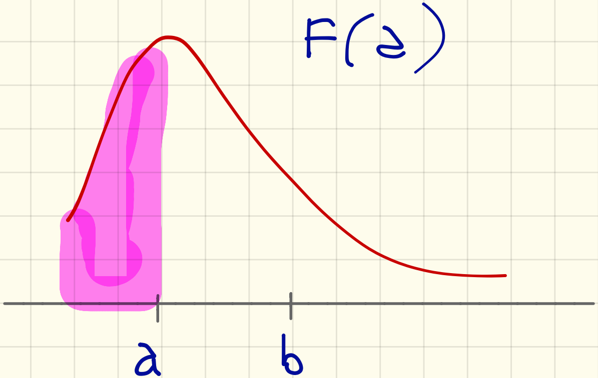
$$P(a < X \leq b) = P(X \leq b) - P(X \leq a)$$
$$= F(b) - F(a)$$


a
b

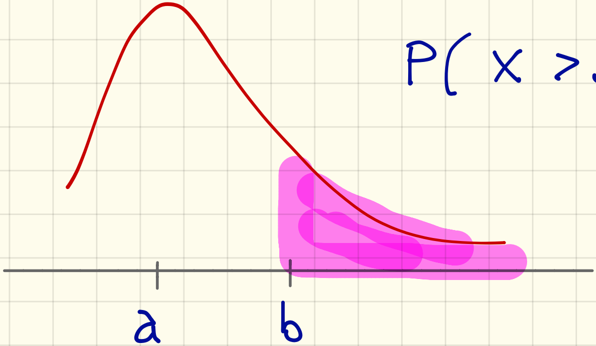
=


F(b)
a
b

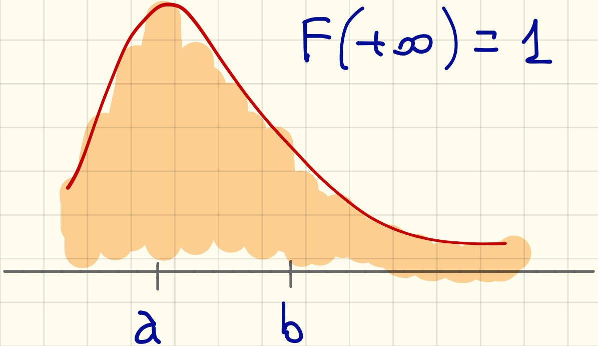
−


F(a)
a
b

$P(X > b)$

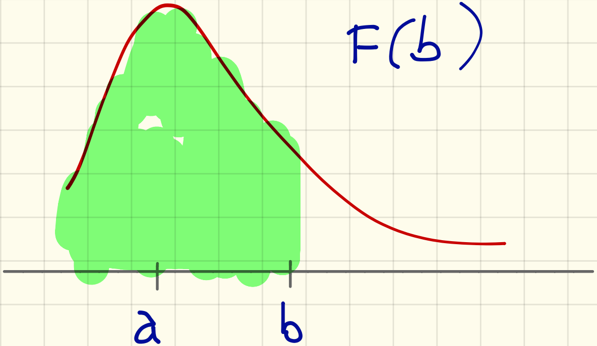


$=$



$F(+\infty) = 1$

$-$



$F(b)$

E' possibile esprimere tutte le probabilità associate ad un modello probabilistico in termini di funz. di ripartizione

particolarmente utile nel caso continuo

$$\boxed{F(x) = P(X \leq x)} = P(X < x)$$

Analogamente a quanto visto per la funz. di ripartizione empirica (prima parte del corso), anche per la f. r. teorica si ha:

$$\lim_{x \to -\infty} F(x) = 0 \qquad \lim_{x \to +\infty} F(x) = 1$$

Funzione di ripartizione modelli continui

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt$$

risolvibili in forma chiusa

ragionamento geometrico

$\boxed{\text{V.C. UNIFORME CONTINUA}}$

soluzione integrale

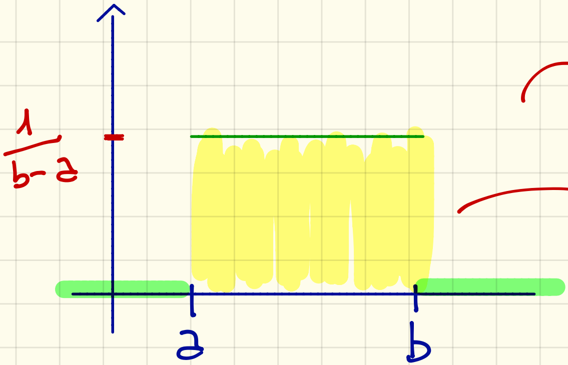
$\boxed{\text{V.C. ESPONENZIALE NEGATIVA}}$

non risolvibili in forma chiusa

metodi numerici

$\boxed{\text{V.C. NORMALE}}$

v. c. uniforme continua



funz. densità

$$\int_R f(x)\, dx = 1$$

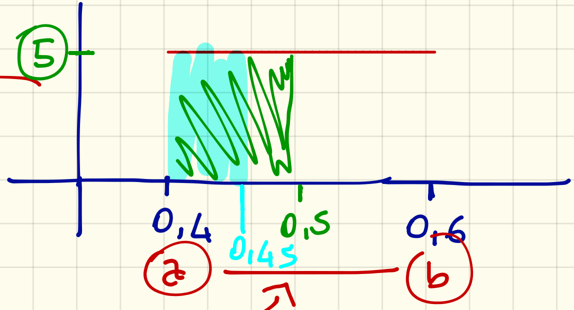
Area rettangolo giallo

AREA = 1 = base × altezza

$$b-a \times \frac{1}{b-a}$$

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{altrove} \end{cases}$$

Esempio $\rightarrow$ macchina dispensatrice bevande



$$f(x) = \begin{cases} \dfrac{1}{0{,}2} = 5 \\ 0 \quad \text{altrove} \end{cases}$$

$$P(X < 0{,}5) = \int_{-\infty}^{0{,}5} f(x)\,dx = \overbrace{(0{,}5 - 0{,}4)}^{\text{base}} \times \overbrace{5}^{\text{altezza}} = 0{,}5$$

$$\frac{a+b}{2} \longleftarrow \begin{bmatrix} \text{media} \\ \text{mediana} \end{bmatrix}$$

$$P(X < 0{,}45) = \int_{-\infty}^{0{,}45} f(x)\,dx = (0{,}45 - 0{,}4) \times 5 =$$

$$= 0{,}05 \times 5 = 0{,}25$$

La v.c. uniforme assume valori tra a e b

la prob. di osservare valori in un dato intervallo dipende solo dalla lunghezza dell'intervallo e non dalla sua posizione

$$P(\underbrace{0{,}4 < X < 0{,}42}_{\text{intervallo ampiezza } 0{,}02}) = P(0{,}47 < X < 0{,}49) = \dots$$

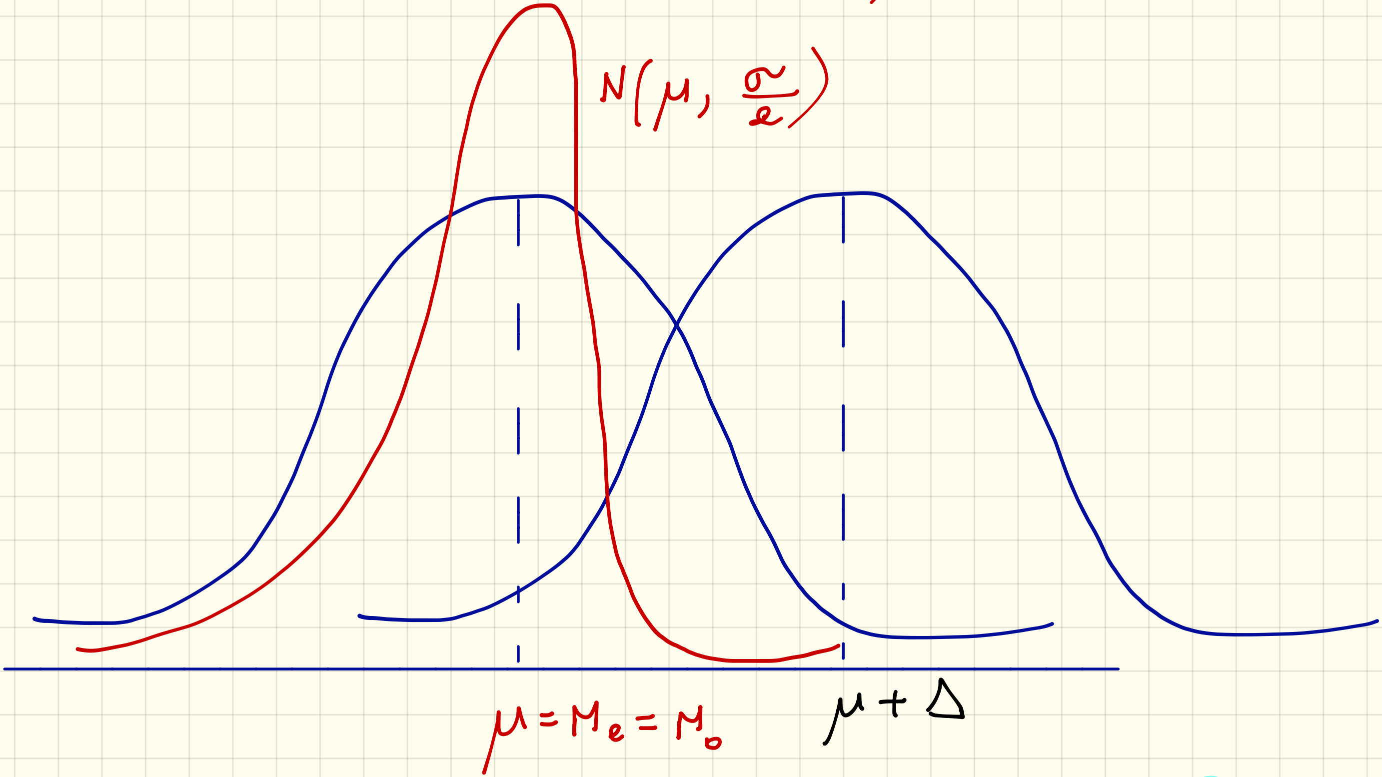
lo stesso vale per qualunque intervallo della stessa ampiezza

$X \sim Unif(a, b)$

$$E(X) = \frac{a+b}{2}$$

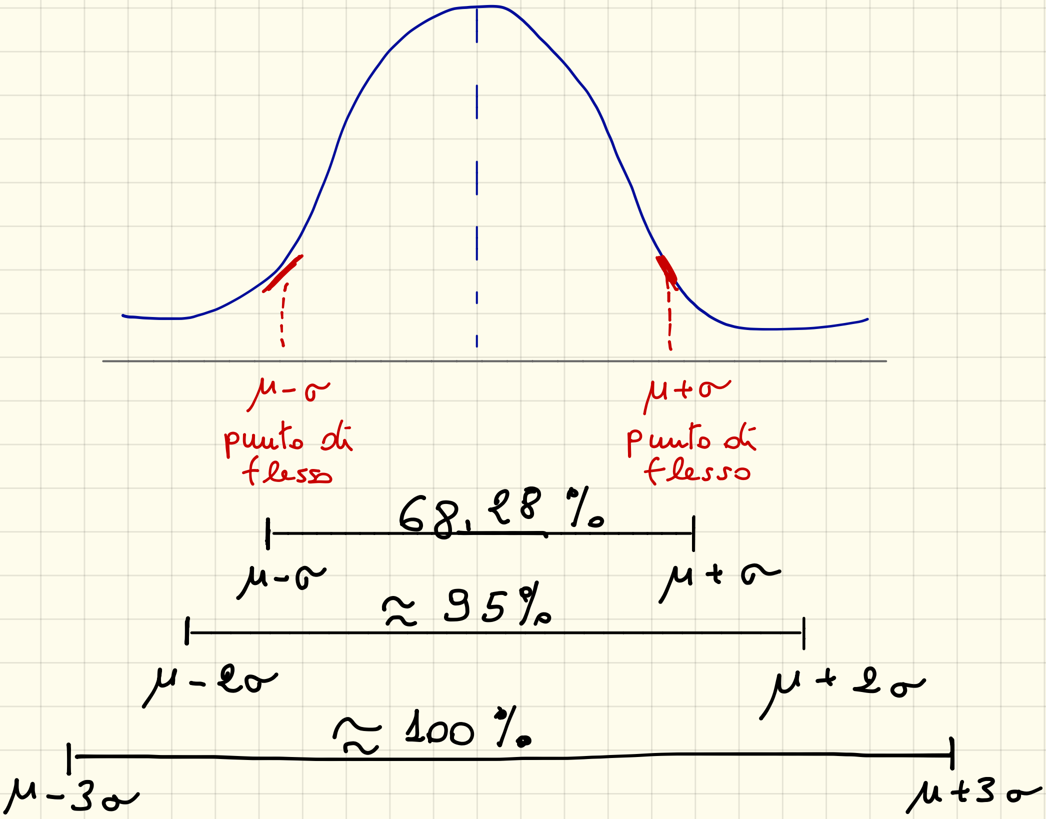
$$Var(X) = \frac{(b-a)^2}{12}$$

V.C. NORMALE (O GAUSSIANA)



$$X \sim N(\mu, \sigma^2) \rightsquigarrow f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

media varianza



Anche se teoricamente $X \sim N(\mu, \sigma^2)$ può assumere valori su $\mathbb{R}$, in realtà la quasi totalità dei valori si trova in $[\mu - 3\sigma, \mu + 3\sigma]$

La v.c. normale è importante per due motivi:

- può essere utilizzata per descrivere molti fenomeni reali associati a processi di misurazione
- la combinazione lineari di fenomeni descritti usando altri modelli probabilistici, sotto particolari e non troppo restrittive condizioni, può essere approssimata usando la distribuzione normale → TEOREMA LIMITE CENTRALE

esempio: vedi il comportamento della $\underbrace{\text{v.c. binomiale}}_{\text{somma di } n \text{ v.c. Bernoulli indipendenti}}$ al crescere di n

Per calcolare le probabilità associate ad una curva normale dovrei risolvere questo integrale:

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

funzione di densità di probabilità

non ha una forma analitica chiusa

metodi numerici

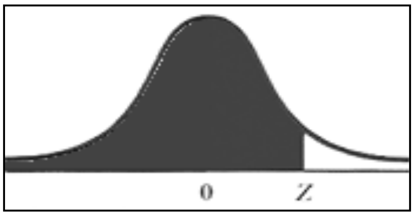
sfruttare le tavole della funz. di ripartizione

caso particolare v.c. normale standardizzata

[soluzione di questo integrale per $Z \sim N(\mu = 0, \sigma^2 = 1)$)

$Z \sim N(\mu=0, \sigma^2=1=\sigma)$

Tavola della distribuzione Normale Standardizzata



Z	Funzione di ripartizione della normale standardizzata									
Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

PROPRIETA' DELLA V. C. NORMALE (1/2)

una trasformazione lineare $Y = a + bX$

di una v.c. $X \sim N(\mu_x, \sigma_x^2)$ è ancora normale

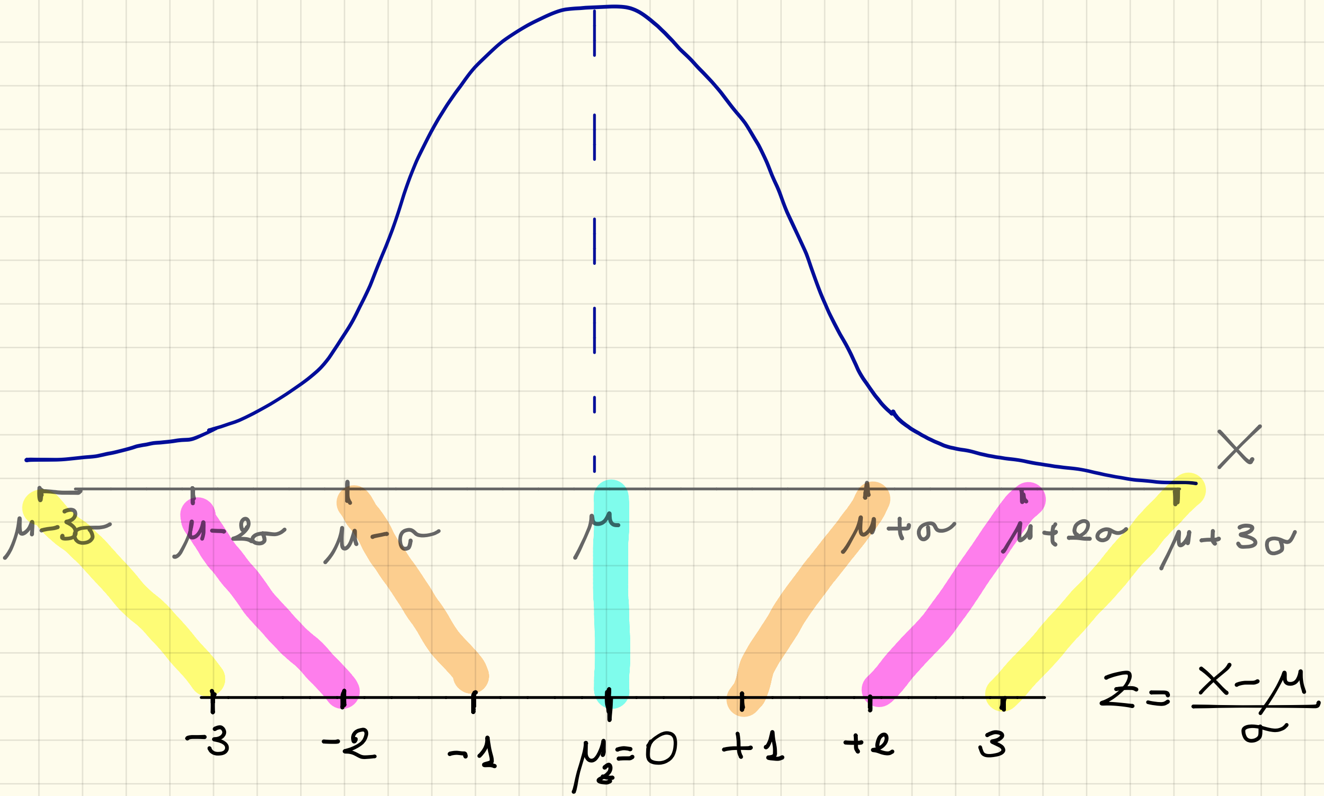
dove $Y \sim (a + b\mu_x \,,\, b^2 \sigma_x^2)$

standardizzazione: particolare trasformazione lineare

$$\underbrace{X \frac{1}{\sigma_x}}_{b} - \underbrace{\frac{\mu_x}{\sigma_x}}_{a} \leadsto \left[\begin{array}{l} a = -\frac{\mu_x}{\sigma_x} \\ \\ b = \frac{1}{\sigma_x} \end{array}\right] \leadsto Y = \frac{X - \mu_x}{\sigma_x} = Z$$

$$Z \sim N(0, 1)$$

standardizzando la v.c. $X \sim N(\mu_x, \sigma_x^2)$ potete fare riferimento ad un'unica tavola



Ad esempio:

$$P(\mu-\sigma < X < \mu+\sigma) \equiv P(-1 < Z < +1)$$

$X \sim N(180, 16)$ — altezza in cm; $\mu = 180$, $\sigma^2 = 16$

funz. di ripartizione

AREA A SX

ascissa $> \mu$

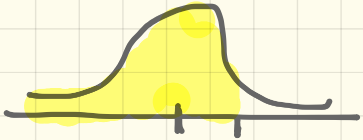
① $P(X < 184)$ $[= P(X \leq 184)]$

$$\underbrace{P(X < 184)}_{F_X(184)} = \int_{-\infty}^{184} \underbrace{f(x)}\,dx = P\Bigg(\underbrace{\frac{X-\mu}{\sigma}}_{Z} < \frac{184-\mu}{\sigma}\Bigg) =$$

$\mu = 180$, $\sigma = 4$

↳ densità $N(180, 16)$

ascissa corrispondente a 184 sulla scala Z



$$= P(Z < +1) = F_Z(+1) = 0{,}8413$$

TAVOLE

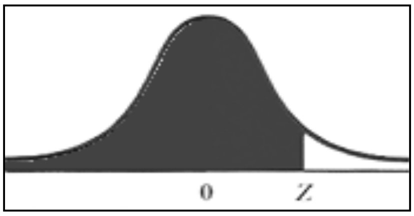
$$P(X < 187) = P\left(\frac{X-\mu}{\sigma} < \frac{187-180}{4}\right) =$$

$$= P\left(Z < \frac{7}{4}\right) =$$

$$= P(Z < +1{,}75) = \underline{0{,}9599}$$

TAVOLE

Tavola della distribuzione Normale Standardizzata



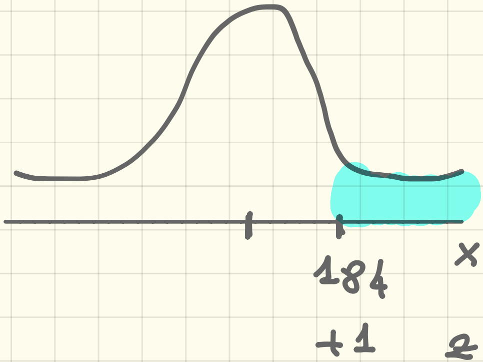
	Funzione di ripartizione della normale standardizzata									
Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

② $P(X > 184) = P\left(\frac{X-\mu}{\sigma} > \frac{184-180}{4}\right) =$

AREA A DESTRA

ascissa $> \mu$

$z > 0$



$= P(z > +1) =$

$= 1 - P(z < +1) =$

$= 1 - F_z(+1) =$

TAVOLE

$= 1 - 0{,}8413 =$

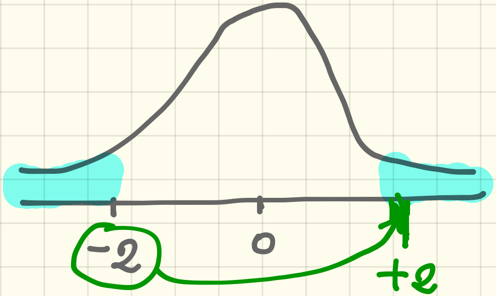
$= 0{,}1587$

3) $$P(\underbrace{X < 172}) = P\left(\frac{X-\mu}{\sigma} < \frac{172-180}{4}\right) =$$

AREA A SINISTRA
(funz. di ripartizione)

ascissa $< \mu$

$z < 0$



$$= P(Z < -2) =$$

$$= P(Z > +2) =$$

caso precedente

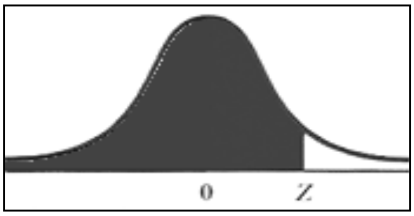
$$= 1 - P(Z < +2) =$$

$$= 1 - F_Z(+2) =$$

$$= 1 - 0,9772 =$$

$$= 0,0228$$

Tavola della distribuzione Normale Standardizzata



Z	Funzione di ripartizione della normale standardizzata									
Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

④ $P(X > \underbrace{172}_{\text{ascissa} < \mu})$

AREA A DESTRA

$\underbrace{\text{ascissa} < \mu}_{z < 0}$

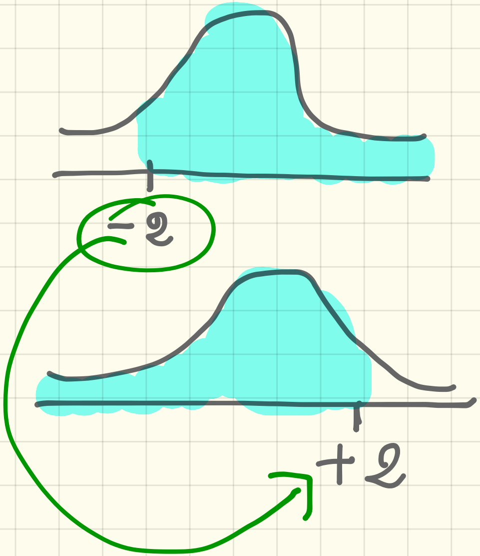
$$= P\left(\frac{X-\mu}{\sigma} > \frac{172-180}{4}\right) =$$

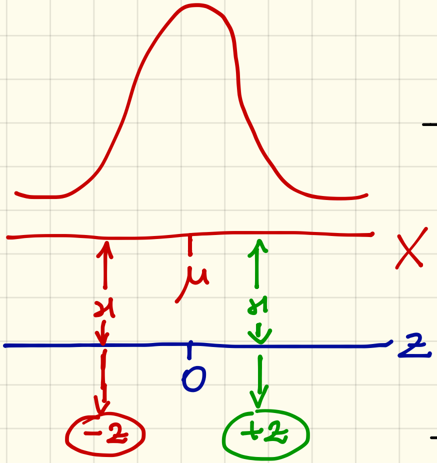
$$= P(z > -2) =$$

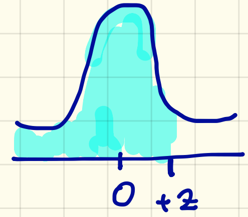
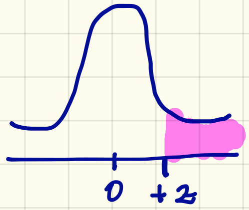
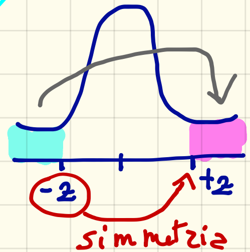
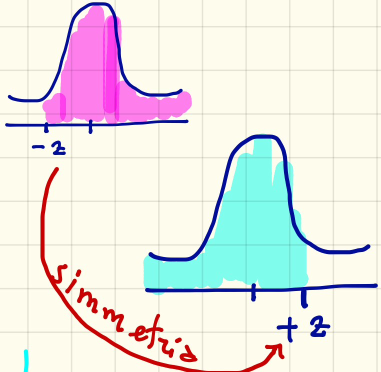
$$= P(z < +2) =$$

$$= F_z(+2) =$$

$$= 0,9772$$





	AREA A SX	AREA A DX
$x > \mu$ $z > 0$	 $F(x) = F(+z) =$ TAVOLE	 $P(Z > +z) = 1 - F(z)$ $= 1 -$ TAVOLE
$x < \mu$ $z < 0$	 $P(Z < -z) = P(Z > +z)$	

$P(a < X < b) =$

$= F(b) - F(a)$

applicate la stessa regola due volte

CALCOLO PERCENTILI V.C. NORMALE

$$X \sim N(\mu = 180,\ \sigma^2 = 16)$$

Determinare i tre quartili di X

$$q_2 = M_e = 180 = \mu$$

$q_3 \rightsquigarrow$ quel valore che lascia a sx il 75%
e a dx il 25%

$\Downarrow$

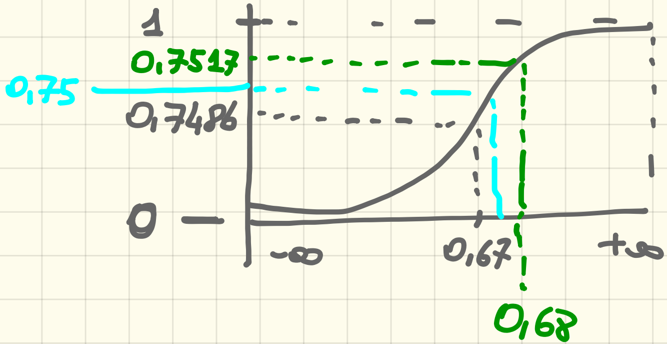
$$F_X(q_3) = P(X \leq q_3) = 0{,}75$$

problema inverso:
nota l'area
bisogna calcolare
la corrispondente
ascissa

$$F_X(q_3) = \underbrace{P(X \le q_3)} = 0{,}75$$

$$F_Z(q_3^z) = P\left(\frac{X-\mu}{\sigma} < \frac{q_3 - 180}{4}\right) = 0{,}75$$

TAVOLE



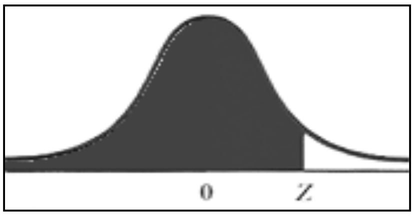
$$F_Z(0{,}675) \approx 0{,}75$$

$$\frac{q_3^* - \mu}{\sigma}$$

$$z = \frac{x-\mu}{\sigma} \Longrightarrow x = \mu + z\sigma = 180 + 0{,}675 \times 4$$

$$= 182{,}7$$

Tavola della distribuzione Normale Standardizzata



	Funzione di ripartizione della normale standardizzata									
Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$q_1 \rightsquigarrow F_X(q_1) = \underbrace{P(X \leq q_1)} = 0{,}25$

$$F_Z(q_1) = P\left(\frac{X-\mu}{\sigma} < \frac{q_1 - 180}{4}\right) = 0{,}25$$

sfruttando la simmetria (e il fatto che la Z è centrata sullo 0)

$q_1^Z = -q_3^Z$

$-0{,}675 \rightsquigarrow q_1^X = \mu + z\sigma = 180 - 0{,}675 \times 4 =$

$= 177{,}32$

$$z = \frac{X - \mu}{\sigma} \rightsquigarrow x = \mu \pm z\sigma$$

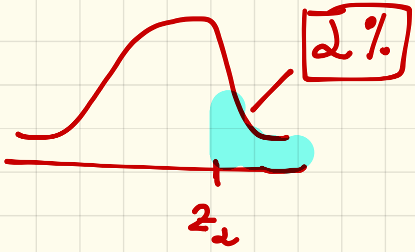
percentili a dx della mediana $(=\mu)$

percentili a sx della mediana $(=\mu)$

$\boxed{\text{NOTA}}$

Per lo svolgimento di molti problemi di inferenza è spesso richiesto il calcolo di alcuni percentili "estremi"

$z_\alpha \rightarrow$ percentile che lascia a dx l'α% dei valori $\rightarrow$



$\alpha = 0{,}1 \quad ; \quad \alpha = 0.1/2 = 0{,}05$

$\alpha = 0{,}05 \quad ; \quad \alpha = 0{,}05/2 = 0{,}025$

$\alpha = 0{,}01 \quad ; \quad \alpha = 0.01/2 = 0{,}005$

PROPRIETÀ DELLA V.C. NORMALE (2/2): PROPRIETÀ RIPRODUTTIVA

Siano:

$$X_1 \sim N(\mu_1, \sigma_1^2) \qquad e \qquad X_2 \sim N(\mu_2, \sigma_2^2)$$

e siano inoltre X_1 e X_2 indipendenti, allora la combinazione lineare:

$$Y = a_1 X_1 + a_2 X_2$$

è una v.c.

$$Y \sim N\Big(\underbrace{a_1 \mu_1 + a_2 \mu_2}_{\mu_Y} \,;\, \underbrace{a_1^2 \sigma_1^2 + a_2^2 \sigma_2^2}_{\sigma_Y^2}\Big)$$

Lo stesso risultato vale nel caso di n v.c.

$X_i \sim N(\mu_i, \sigma_i^2)$; indipendenti

$i = 1, n$

$$Y = \sum_i a_i X_i \longrightarrow Y \sim N\left(\sum_i a_i \mu_i ; \sum_i a_i^2 \sigma_i^2\right)$$

combinazione lineare
di n v.c. normali indipendenti

due particolari
combinazioni lineari

$a_i = 1$
$\forall i = 1, n$ — v.c. SOMMA

$\mu_Y = \sum_i \mu_i \qquad \sigma_Y^2 = \sum_i \sigma_i^2$

$a_i = \frac{1}{n}$
$\forall i = 1, n$ — V.C. MEDIA $\longrightarrow$ RISULTATO RILEVANTE PER INFERENZA SULLA MEDIA

$$\mu_Y = \frac{\sum_i \mu_i}{n} ; \quad \sigma_Y^2 = \sum_i \sigma_i^2 / n^2$$